

# Mathematics in the School Curriculum: an International Perspective

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## Introduction

The mathematics thematic study was designed to enrich the *Archive* by examining:

- curriculum organisation and structure
- assessment arrangements
- classroom organisation and teaching methods
- use of textbooks and other resources, and
- teacher specialisation.

These aspects of mathematics, for students aged five to 14, are used to structure the study, which incorporates material from the *Archive*, from specific enquiries about mathematics to the 16 countries of the study, from discussion at the international seminar on mathematics in the school curriculum held by QCA in February 1998 and from published sources such as the Third International Mathematics and Science Study (TIMSS) and others (see Appendix 1). Practice in the 16 countries studied is outlined, although complete information is not available for all of them. References are to the 16 countries in the *Archive* and cannot be generalised beyond these.

Basic details of the structure of the education systems in the countries studied are provided in Table 1, Appendix 2, which indicates:

- compulsory education (starting age, duration)
- educational phases (broadly categorised as pre-school, primary, lower secondary and upper secondary).

## Curriculum organisation and structure

In this report, the 16 countries of the study are divided into two groups:

Group A: Centralised Government: England, France, Hungary, Italy, Japan, Korea, The Netherlands, New Zealand, Singapore, Spain, Sweden

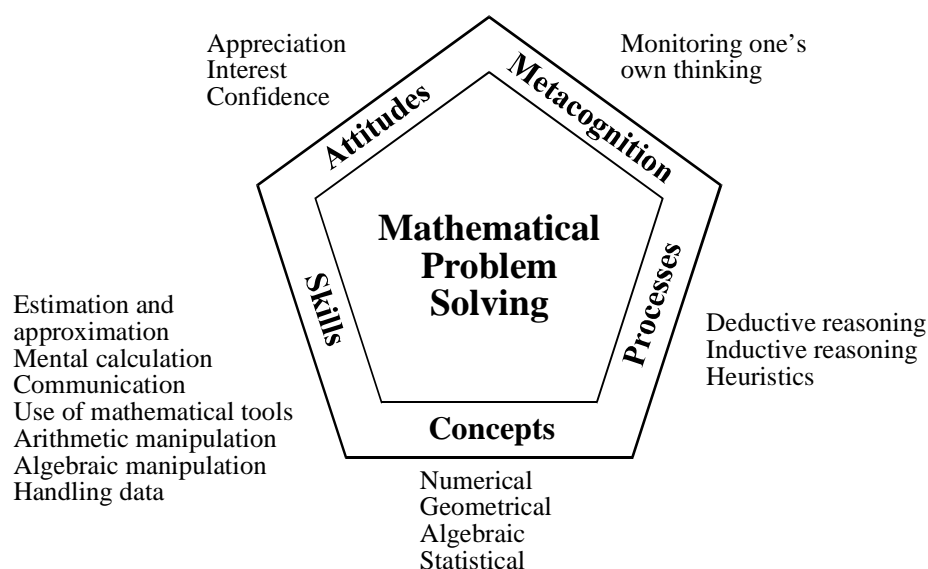
Group B: Federal Government: Australia, Canada, Germany, Switzerland, USA

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There are considerable differences between the two groups in their organisation of mathematics education for ages five to 14, but regional flexibility and differences in local implementation are not unique to federal states.

### **A nationally or regionally defined mathematics curriculum and its content areas**

All of the 11 countries in Group A have some form of compulsory national curriculum for mathematics. In most cases, this runs from the start of compulsory education (see Appendix 2) and continues to beyond age 14. Practice varies from a highly specified curriculum, as in Singapore, to more flexible arrangements with considerable local autonomy. A diagram summarising the framework used in Singapore is shown below as Figure 1. The emphasis on problem solving is of interest, and reflects concern in Singapore about the importance of this.



Hungary, Italy, and Spain stress local implementation and adaptation, while Japan issues national guidelines. Korea also stresses local implementation, but the curriculum is detailed by grade within topics, which are further subdivided. In Sweden the system includes targets for minimum attainment.

In the federal states of Group B, education is the responsibility of the regions, and a national mathematics curriculum as such can only exist provided the regions agree to it both in principle and in practice.

Canada, Switzerland and the USA do not appear to have developed centralised guidance, but Australia provides an interesting example of a federal state which has developed national, non-statutory, guidance for mathematics.

The 'National Statement on Mathematics for Australian Schools' provides a rationale - 'Principles for School Mathematics' - in the context of modern society. It also includes a section on 'The Scope of the School Mathematics Curriculum' which 'describes mathematical understandings, skills, knowledge and processes which should typically be made available to

students'. The stated purpose of the document is to 'provide a framework around which systems and schools may build their mathematics curriculum' and it states that it is 'descriptive rather than prescriptive'.

The different ways in which systems break down mathematics into content areas is analysed in Table 2 which follows. Division of the mathematics curriculum into content areas is fairly consistent in the examples available for study, although there are variations in terminology, and not every system includes each content area for particular age groups. Probability, for example, is absent from several curricula.

The basic content building blocks of the curricula studied are

- Number
- Algebra
- Geometry
- Measures
- Probability
- Statistics.

The combination of Number and Algebra is confined to England and Hungary; other systems separating these aspects of mathematics. Several systems specify Algebra in more detail. It should be noted that use of the same heading does not imply that the approach to the content concerned is uniform, and countries vary, for example, in what they view Geometry as comprising.

Measures is the subject of some variation as to whether it is combined with another area or kept separate. Where Measures is combined, the other area of mathematics chosen is Geometry, as in England. None of the systems available for study have opted for combining Measures with Number, which is rather surprising given that aspects of Measures can be clearly seen as relating to Number rather than Geometry. The treatment of Measures in Singapore is unusual, being combined with Money and Mensuration for Primary 5 (age ten to 11). While Money is a form of measure, it is more commonly subsumed in Number and Mensuration is usually seen as part of Geometry, although aspects of Measures (and Number) are clearly involved.

The Singapore system as a whole has another unusual feature, using different topic headings for different age groups to a greater extent than other systems. This may well derive from the approach to differentiation in Singapore outlined in further detail below. Singapore also specifies the primary curriculum in a greater number of content areas than usual; other systems tend to use subdivisions for this level of detail.

The combination of Probability and Statistics is frequently noted. The only elements in national curricula which do not easily fit into the categorisation in Table 2 are the Attitudes and Appreciations section of the Australian National Statement, which is also present in the Singapore framework, Logic in the Italian primary school curriculum and the Quantitative Relations element of the Japanese lower secondary curriculum which includes function with

probability and statistics. This unusual combination reflects a somewhat different view of the relationships within mathematics.

**Table 2: Organisation of National Curricula by Area of Mathematics**

Area of Mathematics	Problem Solving/Process	Number	Algebra	Geometry	Measures	Probability	Statistics
<b>England</b>	Using and Applying Mathematics	Number and Algebra		Shape, Space and Measures		Handling Data	
<b>Hungary</b>	Developing Mathematical Thinking	Arithmetic, Algebra, Sequences, Functions		Geometry, Measures		Probability, Statistics	
<b>Italy</b> <b>Elementary</b> <b>Lower Secondary</b>	Logic	Arithmetic	Algebra (Linear Eqns, Problems)	Geometry & Measures		Probability, Statistics and Data Processing	
		Number Sets (Natural and Real)		Geometry Transform's; Co-ordinates	Probability		
<b>Japan</b> <b>Lower Secondary</b>		Numbers and Algebraic Expressions §		Geometrical Figures		Quantitative Relations (§ including Function)	
<b>Korea</b> <b>Primary</b> <b>Middle</b>		Number; Operations	Equations & Inequalities Functions	Geometric Figures	Measurement		Relations
		Number & Expression;		Geometric Figures		Statistics	
<b>Singapore</b> <b>Primary 5</b> <b>Secondary 1</b>	Problem Solving	Whole Nos.; Fractions; Decimals; Direct Prop.; Percentages	Algebra	Geometry	Money, Measures and Mensuration		Statistics; Average and Rate
		Arithmetic		Geometry; Mensuration		Statistics	
<b>Australia*</b> <b>(National Statement)</b>	Mathematical Enquiry Choosing and Using Maths	Number	Algebra	Space	Measurement	Chance and Data	
<b>Alberta (Canada)</b>		Number (Concepts; Operations)	Patterns and Relations (Patterns, Variables and Equations; Relations and Functions)	Shape and Space (Measurement; 3-D Objects and 2-D Shapes; Transformations)		Statistics and Probability (Data Analysis; Chance and Uncertainty)	
<b>British Columbia (Canada)</b>	Problem Solving	Number	Patterns and Relations	Shape and Space		Statistics and Probability	
<b>Wisconsin (USA)</b>	Mathematical Processes	Number Operations & Relationships	Algebraic Relationships	Geometry	Measurement	Statistics and Probability	

\* There are eight headings in the Australian National Statement, the other being Attitudes and appreciations

There is also variation in how mathematical process is viewed, sometimes being regarded as a content area, but otherwise seen as cutting across content areas. This aspect of mathematics is absent from the curricula for Korea and Italy, but is present in Alberta (Canada), but with a different status from the mathematical areas listed in Table 2. In Singapore, this aspect of mathematics is central to the curriculum framework, but is not used as a content heading in the later primary years.

The apparent consistency shown in the curricula available for study is interesting, but should not be over-interpreted. In one sense these common building blocks reflect the nature of mathematics itself, but more particularly they reflect the nature of school mathematics as seen in the different systems. They can also be seen as a communication device: a way of structuring the subject to communicate the curriculum to teachers and to break down what would otherwise be a large mass of material. It is also possible that the commonality is due, in some part, to systems studying the curricula of others when revision is being undertaken. This seems probable since many systems have recently revised their curricula, sometimes as part of a regular review process.

The evidence available suggests that, at this macro level, there is considerable agreement on those areas of mathematics which should comprise the mathematics curriculum from age five/six to 14. Beyond this, however, there are likely to be considerable differences between systems in the balance between these mathematical areas and in the degree to which particular aspects of mathematics are stressed and how they are approached. There is also the issue of how compulsory the national or regional curriculum is; in some countries compulsion is not always possible or desired. In practice, the operation of a national curriculum depends in part on the degree of local autonomy given to schools and districts. This in turn governs the extent to which a curriculum can be made compulsory or is for guidance with local adaptation. Such local adaptation makes generalisation as to emphasis hazardous; what seems to be the case on paper may not be so in the classroom.

### **Performance standards**

Performance standards of some sort are not a necessary part of a national curriculum, which may merely list the content to be covered. In such cases there may be some qualifying phrases which indicate the difficulty/complexity level expected, but standards are not explicitly defined, although this may be addressed by distributions of examination or test grades. The performance standards available for review fall into two types: those which are made up of a series of levels which are not directly related to age, and those which consist of age-related targets for some or all year groups or grades. The former are essentially descriptive in nature, defining a progression of descriptions of performance, while the latter, although descriptive, are normative in nature, being intended to define expectation.

In practice, the distinction between the two types can be blurred by adding to non age-related performance standards expectations of the distribution of levels expected to be awarded at particular ages or grades or by a subsequent direct matching of levels to ages or grades. Examples of both methods of adding a normative emphasis to descriptive levels have been identified.

The National Statement for Mathematics in Australian Schools provides one example of a set of descriptive levels. Eight levels of achievement are defined, but these are not related to age or year group/grade. These levels are defined by statements for each strand of learning. They include pointers on how students may demonstrate achievement, and annotated work samples for each level. Levels are defined for:

Working mathematically  
Space  
Number  
Measurement  
Chance and data  
Algebra.

The descriptions include qualifying phrases such as 'simple' 'common' 'with whole numbers' and 'straightforward', which help to define the difficulty level intended. There are also indications of sample performance at each level. The levels are also summarised with general descriptions of the approaches typical of the level.

Examples of the statements for Written Computation, part of Number, are:

Level 2 Uses a variety of strategies, including regrouping, to assist in adding and subtracting whole numbers when unable to complete mentally.

Level 3 Uses understood written methods to add and subtract any whole numbers and amounts of money and to multiply and divide whole numbers by whole numbers to ten.

Level 4 Uses understood written methods to add, subtract, multiply and divide whole numbers, money and measures (two places, whole number multipliers and divisors to ten).

Level 5 Uses understood written methods to add, subtract, multiply and divide whole numbers and common and decimal fractions (whole number multipliers and divisors).

Level 6 Uses understood written methods to calculate with decimal and common fractions and integer powers.

Beyond Level 6, Written Computation is combined with other aspects of Number:

Level 7 Undertakes efficient computations on positive and negative numbers of any size, including rearranging formulae and quoting results to a suitable level of accuracy.

Level 8 Searches for and uses representations for number and operations that will assist the solution of problems by highlighting patterns in numbers or by reducing complexity and computational load.

These statements are similar to the former ‘statements of attainment’ in earlier versions of the English National Curriculum, and are not unlike phrases used in the current ‘level descriptions’ in England. Other parallels include restrictions on the size of number at the lower levels and more global, less well-defined statements at the higher levels.

Reactions from the Australian states to the National Statement have varied. New South Wales has not taken up this initiative, Tasmania has adopted it unchanged, while Victoria has mapped the levels to ages. Victoria has thus introduced a normative element directly rather than, as in the English National Curriculum, where the levels are criterion referenced and the normative element is added via expectations of distributions of levels to be achieved at particular ages. In England, the average seven-year-old, for example, is expected to achieve level 2.

An example of a system which has inbuilt expectations is taken from one of the states in the USA, Wisconsin. Here, performance standards are grade (year group) related and are given for grades 4, 8 and 12 (ages ten, 14 and 18). Examples for Computation are given for grades 4 and 8:

Grade 4 In problem-solving situations involving whole numbers, select and efficiently use appropriate computational procedures such as

- recalling the basic facts of addition, subtraction, multiplication, and division
- using mental math (eg  $37 + 25$ ,  $40 \times 7$ )
- estimation
- selecting and applying algorithms for addition, subtraction, multiplication, and division
- using a calculator

In problem-solving situations involving money, add and subtract decimals

Grade 8 Perform and explain operations on rational numbers (add, subtract, multiply, divide, raise to a power, extract a root, take opposites and reciprocals, determine absolute value)

In problem-solving situations, select and use appropriate computational procedures with rational numbers such as

- calculating mentally
- estimating
- creating, using and explaining algorithms
- using technology (eg scientific calculators, spreadsheets)

The model used in Wisconsin covers only three grade levels and defines performance less specifically than the Australian or English models. It is interesting to note that some school districts in Wisconsin have developed the model to provide standards for other grades, for example grade 6.

British Columbia (Canada) provides performance standards for several grades. Extracts from that for grade 8 (age 14) are shown below:

It is expected that students will apply the four basic arithmetic operations to rational numbers to solve real-world problems.

It is expected that students will: estimate, compute and verify the sum, difference, product, and quotient of rational numbers

In addition, the material for teachers suggests assessment strategies and recommends learning resources. Again, the expectation of performance is less detailed than the Australian or English systems, although it bears some resemblance to the more global statements at the higher levels of the Australian series of levels.

In another Canadian province, Alberta, there is a set of standards similar to those in British Columbia. Taking Number Operations at grade 8 (age 14) as an example:

General Outcome: Apply the four basic arithmetic operations to rational numbers to solve problems.

Specific Outcomes: estimate, compute and verify the sum, difference, product, and quotient of rational numbers, using only decimal representations of negative rationals.

The same basic systems have been developed in these two provinces, but there are differences in presentation and terminology and, sometimes subtle, differences in phrasing and emphasis.

In Sweden, another approach to performance standards can be seen in the national tests for grade 5 (age 12). Expectations are expressed in terms of the number of problems answered correctly in the wide-ranging tests. Such criteria as 'Correctly answered at least 8 problems', 'Correctly answered 1a and at least 2 of the others' are used. Performance on test questions or larger-scale pieces of student work is used to define the standards in this context. Sweden also identifies 'goals that should be achieved' and 'goals to strive for', an interesting combination of minimum competency and more ambitious targets.

The nature of performance standards varies in generality between the systems available for study. The examples covering Number Operations illustrate this well, although none could be described as global in nature. The detailed standards specify the operations concerned, and often (attempt to) indicate the level of difficulty by specifying the nature and size of the numbers to be operated on. References to method, written, mental, etc. are common.

The standards in some systems are more general than in others, but it seems likely that these are, in practice, made more specific by support materials, custom and practice, assessment systems or a combination of these. The issue of specificity has been tackled in all the performance standards available for study by breaking mathematics down into manageable pieces with defined boundaries. Even the more global statements apparent at the higher levels can be seen as resulting from the absence of restrictions, for example on the size of number, present at the lower levels.

There are interesting differences in the structures into which the detailed standards fit. There are examples of systems of levels, with or without direct relationship to expectation at

particular ages. These contrast with sets of standards for each year, or for particular key points in the system concerned, into which expectation is firmly built and achievement of it easily judged from assessment results. Standards for particular year groups exhibit the dangers of any minimum competency model; that teachers may emphasise getting the maximum number of pupils to the set standard, at the expense of taking high attainers further, with the minimum becoming, in effect, the maximum as well.

These dangers are not as pronounced in descriptive level systems, since, even when some relationship to age or grade has been introduced, levels beyond the norm(s) for an age or grade are available for high attainers, and their teachers, to strive for. The systems studied take differing views on the advantages in clarity of performance standards clearly linked to age or grade against systems of levels which do not present the same danger of the minimum becoming the norm.

### **Materials to help teachers understand performance standards**

Performance standards themselves often contain material which is intended to help teachers to understand and operationalise them. The familiar processes of embodying restrictions on size of number involved etc. into the performance standards shown above provide examples of this approach. Some of the curricula which incorporate performance standards are accompanied by extensive documentation for teachers which is separate from the standards themselves. Sample questions or tasks are a common approach, exemplifying what the standard entails and offering a contribution to assessment by the teacher. Hungary, for example, provides examples of probability experiments with coins and dice.

Another common type of material is the provision of samples of students' work annotated to indicate how aspects of the standard have been met. This has been a familiar part of the National Curriculum in England since its early days and, in Australia, all of the states, including Victoria, which have adapted or adopted the National Profiles have chosen this course. In the USA, a range of materials has been developed to assist teachers to understand proficiency levels, with assessments developed in parallel with curriculum materials. The influence of the National Council for Teachers of Mathematics (NCTM) standards is apparent. Annotated work is frequently used where the process elements of mathematics are concerned or where the targets concern extended pieces of mathematics.

Other countries, such as Sweden, offer no such material or, like Singapore, expect teachers to use the syllabus provided and textbooks to internalise the standards. Singapore, however, because of its relatively small size can and does support all teachers via meetings to introduce and implement changes in curriculum, and places particular stress on clarifying the aims of reforms for teachers.

The role of assessment in clarifying the meaning of standards is apparent in several systems, for example in Sweden where detailed scoring instructions for tests have already been discussed. National tests or assessments which address performance standards, whether they are compulsory or optional, also play a role in exemplifying the meaning of the standards for teachers. They do so both by providing examples of questions which address the standard and by quantifying the degree of success on such questions required for the achievement of the standard. The national tests in England provide a good example of this and have shaped

teachers' perceptions and understanding of the curriculum and their own classroom assessment against the same performance standards.

### **How often topics are taught**

It is difficult to establish how often specific mathematical topics are revisited in the 16 countries of the study. A list of content to be taught does not indicate this, but systems with performance standards are more revealing, since it is possible to see the same topic recurring at higher levels in more complex or difficult circumstances. For example, the performance standards outlined above clearly illustrate revisiting for aspects of Computation. The level of complexity increases but the same topic, say multiplication, is revisited in a fashion typical of a spiral curriculum.

The spiral curriculum is the usual model in Australia for the main topics in the curriculum and also in Singapore, where the aim is to cover a topic at an appropriate level which increases in depth with time. In the USA, most new materials adopt a spiral approach, with those funded by the National Science Foundation often revisiting a concept in a different way. The same approach is adopted for some topics in Japan. In England, the spiral curriculum has been the standard model for many years, but topics tend to be revisited more often than appears to be the case in other systems. How often topics are revisited is a matter left to the discretion of the teacher in some systems, such as Sweden, or is decided at school or district level. Even so, some form of spiral appears to be present in all the curricula available for study.

Where textbooks suggest a frequency of teaching for topics, there is no guarantee that this will be reproduced in the classroom, since teachers usually have the freedom to vary their lessons from those suggested by scrutiny of the textbook. It is noticeable, however, that in many systems the textbooks are year group or grade specific and that, in such circumstances, a spiral curriculum in the textbooks will be repeated in the classroom unless the teacher opts to omit whole sections of books and combine material from books intended for different age groups or grades, a somewhat unlikely strategy.

The real differences between curricula may lie in the pitch of the spiral - the degree to which the spiral is flat, with a relatively low rate of revisiting topics, or extended, with topics revisited comparatively frequently. It has not been possible to establish the pitch of the spirals in the curricula as implemented. However, textbooks may shed some light here; certainly textbooks used for a given year group in England would seem to suggest a greater fragmentation of the curriculum and a frequent return to particular topics. Many systems, again from comparison of textbooks, would seem to spend more time on consolidation of a particular level of a topic before a later return to develop the topic further.

### **Hours spent teaching mathematics**

Of the 11 countries of the study with a centralised government, seven indicated specific amounts of time allocated to mathematics. Given the variations between the educational systems, the proportion of time allocated to mathematics was remarkably uniform - usually between 15 and 20 per cent of that available.

There are differences related to age; pupils aged six or seven appear to spend around 19 per cent of their time on mathematics, with all of the allocations falling in the range 15 to 23 per

cent. For students in the middle of the five to 14 age range, the proportion of time spent on mathematics tends to be slightly lower, falling in the range 14 to 21 per cent. For the oldest pupils, the pattern is more varied, with Hungary and Spain allocating lower proportions of time (below 15 per cent) and Singapore allocating between 18 and 23 per cent according to the course taken. The remaining countries' allocations are again between 15 and 20 per cent.

This apparent consistency should, however, be treated with caution, as a difference of five per cent is a considerable amount over one year, and the amount of teaching time per year is not consistent between countries. Nevertheless the predominance of figures in the 15 to 20 per cent range in centralised countries is of interest, since a value judgement that this proportion of teaching time should be allocated to mathematics is apparently frequently made.

An alternative approach to time spent on mathematics is available to supplement the review of 'official' time allocations. The Third International Mathematics and Science Study (TIMSS) gathered data on time spent on mathematics, and the results available for ten-year-olds are shown in Table 3. All of the countries of the QCA study for which TIMSS data are available are included in the Table. Bold text is used to highlight the most common time allocations reported by teachers, and the percentages are of pupils in the schools in the TIMSS samples.

**Table 3: Weekly hours reported spent teaching mathematics: (Grade 4/Age Ten)**

**TIMSS Data**

(% of pupils)

Country	< 2 hours	2 to < 3.5	3.5 to < 5	5 or more	Average (hours)
England	0%	11%	<b>48%</b>	<b>41%</b>	4.6
Hungary	0%	<b>72%</b>	25%	3%	3.3
Japan	1%	6%	<b>92%</b>	2%	3.7
Korea	0%	<b>93%</b>	6%	2%	2.9
Netherlands	0%	5%	<b>39%</b>	<b>56%</b>	4.7
New Zealand	7%	<b>34%</b>	<b>45%</b>	14%	3.6
Singapore	0%	0%	2%	<b>98%</b>	5.5
Australia	0%	24%	<b>37%</b>	<b>38%</b>	4.2
Canada	3%	19%	<b>39%</b>	<b>39%</b>	4.4
USA	12%	9%	<b>33%</b>	<b>46%</b>	4.2

This table reveals several features of interest. Three of the systems with centralised governments (Japan, Korea and Singapore) stand out because of the consistency of the time allocations reported; over 90 per cent of pupils fall in one time category, although a different one in each case. Hungary is the only other system where one category dominates. The other

systems, including the federal states, have considerable proportions of pupils in two adjacent categories.

The average time allocations are also of interest, with Singapore the highest at 5.5 hours and Korea the lowest at 2.9 hours. England lies towards the upper end of the range, at 4.6 hours.

The equivalent data for 14-year-olds in the countries relevant to the QCA study are shown in Table 4 below. For some countries, including England, no data are available.

**Table 4: Weekly hours reported spent teaching mathematics: (Grade 8/Age 14)**

**TIMSS Data**

(% of pupils)

Country	< 2 hours	2 to < 3.5	3.5 to < 5	5 or more
France	2%	10%	87%	2%
Hungary	0%	75%	23%	1%
Japan	4%	91%	4%	0%
Korea	1%	90%	5%	5%
Netherlands	3%	97%	0%	0%
New Zealand	5%	42%	50%	3%
Singapore	0%	52%	48%	0%
Spain	2%	28%	62%	8%
Sweden	3%	97%	0%	0%
Australia	5%	50%	44%	1%
Canada	3%	31%	50%	17%
Switzerland	2%	14%	71%	13%
USA	8%	24%	58%	11%

The tendency amongst countries with a centralised government is for one category to dominate, although there are exceptions, Singapore and New Zealand in particular. France, Japan, Korea, the Netherlands and Sweden indicate a high degree of consistency between schools. That Singapore does not is probably due to the differentiated nature of time allocations for older pupils of different attainment levels, which is a feature of the Singapore system.

As would be expected from the officially reported time allocations, those reported for older pupils are lower, although the TIMSS report does not give averages for this age group to quantify this.

Taken together, Tables 3 and 4 reveal that the actual variation in teaching time for mathematics is considerable, and that comparatively few systems show a very high degree of consistency between schools. The tables also suggest that 'official' time allocations need to be treated with caution in most systems. The value judgements behind the 'official' time allocations do not always seem to be shared by schools.

### **Emphasis on particular areas of mathematics**

No evidence is available to suggest that specific time allocations for different areas of mathematics are in use in the 16 countries of the study. In Australia, for example, the six content areas of the National Statement on Mathematics for Australian Schools (Working Mathematically, Space, Number, Measurement, Chance and Data and Algebra) cannot be assumed to be given equal time allowances. In Victoria more time is spent on Number with young children and more time on Algebra with older students. This matches the situation in England, where the emphasis within the Number and Algebra attainment target is on Number for younger pupils but progressively switches to Algebra for older pupils.

In the USA, integrated approaches to mathematics are breaking up the traditional divisions into Arithmetic (often taught for eight years), Algebra (one year) and Geometry (one year). This point may well be a salient one, since division of time between topics implies strong division lines between topics which may well not be consistent with the general curriculum approach to mathematics.

The 16 countries studied were also asked to indicate any particular emphasis within their curricula. Australian curricula are reported to stress real world contexts with a lack of emphasis on proof. This latter point could also be applied to England, while in the Netherlands the emphasis on real world problems drives the curriculum.

### **Assessment arrangements**

In contrast to time allocated to mathematics, assessment arrangements show considerable variation. Table 5 indicates the formal assessment arrangements in each system.

**Table 5: Summary of formal assessment arrangements – Ages Five to 14**

Country	Compulsory National Tests	Optional National Tests	National Tests on Samples or Surveys	School Leaving Examinations
England	Ages 7, 11, 14	Ages 8, 9, 10		
France	Ages 8, 11			
Hungary			Various ages	
Italy				Ages 11, 14
Japan			Periodic	
Korea			Ages 10, 11, 12	
Netherlands		Age 12		
New Zealand		Age 12	Ages 8,12	
Singapore				Age 12
Spain			Age 12	
Sweden	Ages 9, 12			
Australia	Varies			
Canada	Varies			
Germany				
Switzerland				Age 10-12
USA	Varies		Ages 9, 13	

Table 5 shows that all of the systems with a centralised government have some sort of formal, though not always compulsory, assessment arrangements. Singapore and Italy have primary school leaving examinations which involve mathematics assessment. Italy also has a lower secondary school leaving examination at age 14. The function of the examination in Singapore is selection.

Beyond these, the purpose of the assessment affects the structure adopted. France gives particular importance to diagnostic assessment, with compulsory mass diagnostic assessment every other year for pupils aged eight and 11 (e.g. 11-year-olds in 1996 and eight-year-olds in 1997). This has replaced, partly on economic grounds, the system where both age groups were tested every year, and incorporates optional assessment in the non-compulsory years. The assessment of 11-year-olds coincides with a change of school, that is to say entry to lower secondary education in the *collège*. The system is unusual in that testing takes place at the beginning of the school year and looks back over the work carried out in the previous year, being intended to inform future teaching. The stress is clearly on formative, rather than summative, purposes.

Other systems which share this emphasis on diagnostic assessment are Sweden, at ages nine and 12, and Queensland (Australia) during the second year of education. In Victoria

(Australia), there is concern that assessments designed to be diagnostic can, in practice, be poor indicators.

Sweden has national tests for students aged 12, and these illustrate that perceptions of purpose may differ. The Government sees the role of these tests as supporting teachers and influencing the allocation of funding to pupils who do not pass, while the National Agency for Education (*Skolverket*) stresses diagnosis for the individual pupil and encouraging reflection on teaching by the teacher. There are also concerns in Sweden about tests influencing the curriculum, and the use of item banks to provide tests is seen as a solution. Some of the development work on these is being carried out in cooperation with other Scandinavian countries.

Periodic surveys are used to assess the state of mathematics education in several countries, including Hungary, Japan and the USA. Other systems favouring this approach include Korea, (at ages ten to 12), Spain (at age 12), Queensland, Australia (at ages ten, 12 and 14) and some provinces of Canada (at age 12). Many of these surveys coincide with the end of the primary school phase.

Surveys sometimes involve sampling of pupils, as formerly carried out in England by the Assessment of Performance Unit, which assessed a sample of 2.5 per cent of the chosen age group, and currently by the National Assessment of Educational Progress (NAEP) in the USA. Sampling can also allow a wider selection of the curriculum to be examined, by using one of a number of different tests with each pupil. It also allows more difficult, and therefore expensive to assess, areas of mathematics to be included. Typically this is achieved by testing sub-samples, rather than the whole sample, on more extended or practical tasks which are more expensive to administer. For national surveys, samples of around two to three per cent often suffice. Larger samples may be needed if a representative picture of performance in different regions is required.

The issue of whether it is possible or desirable to make assessments mandatory is an important one. New Zealand has opted for optional testing from nationally developed item banks, complemented by the testing of three per cent samples at ages eight and 12. In Sweden, assessment at age 12 is mandatory for public schools but not for private ones, as are the national tests in England. In the Netherlands compulsion is not legally possible, but national tests are provided for 12-year-olds, at the end of the primary phase, to help make decisions on the type of secondary school course to be taken. About 60 per cent of primary schools make use of these tests.

These variations reflect both differences in education systems and the view taken on the prime purpose of the assessment: diagnosis to help inform future teaching, monitoring the state of mathematics performance, monitoring the performance of individual schools and so on. Practice in England and France, two of the countries with compulsory national tests, provides an interesting contrast. France stresses diagnosis and thus tests at the beginning of the school year and cycle to look forward. In England, the emphasis can be seen as being on measuring the performance of schools at ages seven and 11. Testing is consequently at the end of the school year looking back on infant education and junior education respectively. In both cases,

purpose influences timing and the age groups chosen. Other influences on age group chosen include selection, as in the Netherlands and Singapore.

Other concerns may result in the absence of formal assessment arrangements. In Switzerland, for example, there are concerns that measuring a process might help to destroy it.

The degree to which assessments can serve more than one purpose has always been a contentious issue. Some aspects of the discussion are technical, and systems which emphasise results at the school level, like England, can also produce a national or regional picture by aggregating school results. More arguable is the degree to which assessment systems can successfully combine summative and formative/diagnostic purposes. A commonly held view is that assessment cannot be expected to meet more than one objective properly, but that summative tests can be used to provide some diagnostic information as a secondary purpose. The concerns in Victoria (Australia) that diagnostic tests are often poor indicators have already been noted.

## **Classroom organisation and teaching methods**

Classroom (and/or school) organisation and the curriculum can be strongly linked, and one or both of these two forms of organisation will be where the curriculum is differentiated for pupils of different attainment levels. However, the general picture for classroom organisation in the primary school (usually to age 12) is one of grouping into classes by age for all subjects including mathematics.

This does not, however, rule out grouping by ability *within* the mathematics classroom. In Victoria (Australia), for example, the full ability range is present in most primary classrooms, but grouping of pupils within the classroom varies and there is some grouping by ability. In Canada, there is a strong feeling that pupils should be taught in mixed ability groups for the first few years.

In many of the countries studied, the range of attainment within the class is narrowed by the practice of pupils retaking a year or, less commonly, skipping a year. Countries where repeating a year in the primary school is a feature of the system include France, Hungary, Italy, the Netherlands, Spain (infrequently), Germany and the USA. In addition, pupils in Korea may skip a year.

Streaming is the practice in Singapore after the first four years of schooling (age ten+), three streams being formed. This represents a value judgement that greater differentiation was desirable made as part of the 1991 review of the Singapore system. The current system incorporates different time allocations for mathematics for the different streams (balanced by different times for English and mother tongue languages), some material taught later to lower attainers, and a deliberate policy of not teaching some material to lower attainers. For example, in the 'Normal Academic' stream pupils are only taught to continue a given number sequence, while in the 'Special' and 'Express' streams pupils also learn to generalise sequences in simple algebraic statements. Such material is not in the Normal Technical stream syllabus. The system results in lower attaining pupils having more time to learn a smaller amount of mathematics.

The approach to differentiation in Singapore contrasts sharply with recent practice in Victoria (Australia), where entitlement - that all pupils should have access to the full curriculum - has been the governing principle. In practice this approach has often been softened somewhat by teachers, who have omitted some material for lower attainers. More recent thinking in Victoria represents a move away from strict views on entitlement.

In Hungary, pupils may be grouped into classes by ability. Korea has plans to group pupils by ability for mathematics and also has programmes for the gifted. New Zealand and Sweden describe their primary systems as comprehensive, while in Canada little streaming is reported. In Switzerland, primary pupils are mostly grouped by age, but the view commonly held in England that Swiss classes are predominantly taught using the 'whole class interactive' method is incorrect. Although this method applies to Zürich, a range of practices exists elsewhere.

Japan is unusual with respect to grouping pupils, having mixed ability classes within which pupils are again often in mixed ability groups, which are deliberately designed so that higher attainers assist and encourage low attainers. The same provision is made for all pupils, irrespective of attainment. This philosophy is, however, complicated by the presence of cramming schools (*juku*) which may provide the differentiation which is markedly absent in the official schools.

In secondary education, repeating a year is again common, being a feature of practice in Hungary, Italy (13 per cent of lower secondary pupils in 1997), the Netherlands, Spain, Sweden, Switzerland and the USA (modified by course options). Streaming is the practice in Singapore (three streams), Switzerland, and often in Canada (two streams).

Selective secondary education, with different schools teaching courses leading to different qualifications, occurs in the Netherlands and Germany, the latter sometimes setting as well. In Sweden, setting is the practice in some schools. In New Zealand over-subscribed schools may select by ability and setting is also practised.

In the USA differentiation has typically taken the form of 'tracking', with pupils taking different courses. There are now concerns that this has led to the teaching of conceptual mathematics to higher attainers and to 'skill and drill' for lower attainers. There are parallel concerns that tracking may leave gaps in the basic knowledge and skills of high attainers. There is thus a trend away from tracking, influenced by the National Council for Teachers of Mathematics (NCTM) standards which stress rich mathematics for all pupils.

Strict regulation of how and when mathematics should be taught has not been identified, but this may exist to a fair extent in systems where textbooks dominate the planning of the curriculum. Singapore offers guidelines on teaching and assessment methods. Several other systems offer guidance. Victoria (Australia) recommends the inclusion of cooperative group work, and the agreed Australian practice on calculators is that all grades should use them, with emphases in the early years on number exploration and extending the range of real life problems which can be tackled. Sweden emphasises mental arithmetic. Current Korean guidance encourages schools to break away from what is seen as 'the present emphasis on rote memorisation of fragmentary information'. In Japan, Korea and Singapore there is anxiety

that thinking skills may not be emphasised enough, and the latter stresses problem solving and reasoning in its curriculum framework.

Another aspect of differentiation is provision for pupils with special educational needs. Most systems indicate that there is provision for (severely) handicapped pupils, often in special schools. In the state of Victoria, Australia, for example, all but the most severely handicapped or sick pupils are taught in ordinary schools and an 'aide' is provided to assist in the classroom for various amounts of time during the school week. In Hungary the proportion of pupils in special schools in 1993/94 was around 1.5 per cent for age six, rising to around three per cent for ages eight to 14. Japan, New Zealand, Sweden, Germany and some Canadian provinces have special schools. Singapore allows children to be educated at home. Italy educates pupils with special needs in mainstream classes with some additional staff support.

The issue of differentiation is one which involves a combination of classroom organisation, teaching methods and the nature of the curriculum. Little consensus is apparent, and the extremes are represented by Singapore, with inbuilt streaming and different curricula from a comparatively young age, and the sort of entitlement view prevalent in Victoria (Australia) in the 1980s. In this view Algebra, for example, is seen as a right and not to teach it unthinkable.

Spain has moved to a staged delivery of the curriculum, moving beyond a year by year curriculum to allow a different approach to differentiation.

In many systems, actual practice is somewhat mixed, with a tendency for a variety of factors to influence differentiation. These include resitting or skipping years, stressing only parts of the curriculum to lower attainers, and organisational forms within the classroom. There is a tendency for the degree of differentiation of the curriculum to increase with age, as might be expected since the range of attainment to be dealt with in mainstream schools also tends to increase with age.

It is noted that performance targets and the structure of the curriculum impact on differentiation. Targets which exist at different levels can allow flexibility in differentiation in schools since a match between age and expectation is not necessarily included. On the other hand some curricula, as in Singapore, assume that pupils are taught in separate streams or schools.

It seems that differentiation, the structure of a curriculum and the nature of performance targets are linked in a complex web of interactions with expectation at the centre. The issue of matching the mathematics to the pupil versus entitlement to learn is clearly one for thought, but must be seen against the political and educational desires to raise standards and to make expectations clear.

Conflict between the needs and demands of higher education and what is practicable is not uncommon, as, for example, in Japan, Singapore, Germany and Switzerland.

## **Use of textbooks and other resources**

Two aspects of use of resources are examined, textbooks, as indicated above, and the use of calculators.

### **Textbooks**

Textbooks play an important role in shaping the curriculum experiences of mathematics of pupils in the five to 14 age range. This is particularly apparent in the first few years of formal education, since teachers are usually generalists, rather than mathematics specialists. The study has looked at approval mechanisms, where these exist, and production mechanisms.

Practice as to whether textbooks are produced by, or have to be approved by, national or local (Education) Ministries again varies between countries. In England, Italy, the Netherlands, New Zealand, Victoria and Sweden there is no system for official approval of textbooks. Countries where approval is required are France, Japan, Korea, Singapore, most Canadian provinces, Germany, most Swiss Cantons, and 21 out of the 50 states in the USA. Textbooks are generally approved in Hungary, while in Spain they are produced under the supervision of the Ministry.

Where there is an official approval system, this involves checking that the prescribed curriculum is being followed. In France, textbooks must indicate which class and level they are intended for on the cover or title page. Japan uses a Textbook Authorisation and Research Council to recommend textbooks to the Minister. This comprises university and school teachers and Ministry officials. Korean textbooks fall into three types: those produced by the Ministry, authorised textbooks (authorised by the Ministry) and recognised textbooks (approved by the Ministry), and consequently provide an interesting example of co-existence.

In Spain, teaching materials must be consistent with the aims, content, teaching methods and assessment criteria in the relevant legislation. They must also indicate the level, stage, cycle or school year for which they are intended. In Canada, materials produced by provinces are normally piloted before receiving official sanction. Private sector materials are usually subject to an approval process involving the Ministry. The trend here is away from a single textbook to a variety from which to choose.

In Germany, books must be in line with the principles of the Constitution and Education Acts, compatible with the syllabus and with research findings, adequately bound and, interestingly, the price must be justified. Hungary examines content, technical quality and, again, price. In Singapore, textbooks must adhere to the syllabus, give comprehensive coverage of the topics in the syllabus, adopt a clear and logical presentation of concepts and offer activities to enhance the learning experiences of pupils. Swiss Cantons have the authority to authorise or prescribe textbooks and generally do so for mathematics. In the USA practice varies by state. About half of the states (21 out of 50) recommend textbooks and have a state textbook adoption programme. California and Texas are two of these and, because of their large populations, influence textbooks nationally.

The general tendency is for books to be part of a series with one book per year or grade level. In Korea, the primary texts are based on semesters, thus leading to two books per year. Italy is

unusual in that its primary textbooks cover a range of subjects, mathematics, science, history, geography and social sciences in the same book.

Countries were also asked if their textbooks for the five to 14 age range placed an unusual amount of stress on particular topics. Particular stress, or absence of it, was not reported frequently, but in Victoria (Australia) textbooks are characterised by the use of real world contexts, lack of proof and an empirical approach. In Italy the emphasis at primary level is seen to be on Arithmetic and Geometry, while those for the lower secondary years also stress Algebra.

Comparing textbooks from different countries raises many questions since, while the books reflect the mathematical education culture, it is difficult to establish exactly how they are used in different classrooms. This is particularly so for expository passages. General opinion in the seminar and from questionnaire returns seems to be that such passages are most frequently used to supplement exposition by the teacher. The power of textbooks to set norms and expectations is generally acknowledged, as is the lack of research into textbook effectiveness.

Textbooks are developed by teams of writers in several systems including Singapore, Switzerland, Wisconsin (USA) and England. In some Canadian provinces a different model is used, with commercial publishers developing books to a specification by the province. Piloting of new textbooks is fairly common. In several systems, including Victoria (Australia) and England, commercial publishers have suffered because curriculum change has resulted in books being out of date before the publishers could recoup development costs. Japan provides an example of regular textbook revision on a four-year cycle.

Examples of differentiation in texts vary in nature. In Canada, for example, extension material in older texts tended to be miscellaneous time fillers, while modern texts tend to feature extension problems, say involving three variables rather than the previous two. In the Netherlands, texts sometimes employ differentiation by method, with, for example, more efficient ways of long multiplication taught to high attainers. Dutch texts also feature additional material for lower attainers.

Important issues identified include how easy it is for pupils and teachers to find their way around textbooks, the links between teachers' guides and pupil materials and manageability. It has been noted that North American teachers' guides tend to be large and without guidance on what is important. Swiss and Canadian examples of the effective use of an index have also been noted. Similar guides in England sometimes suffer from not being well related to pupils' texts.

Comparison of textbooks from different countries and within England is one method of finding out the particular characteristics of textbooks and how textbooks reflect the national curriculum. QCA has therefore also funded a two-part study of primary mathematics textbooks from ten countries, including England. These ten countries are a sub-set of the 16 countries of the international review of curriculum and assessment frameworks project.

The study has reported some interesting preliminary findings. National differences between textbooks include an emphasis on pattern spotting in England, on mental work in France,

Singapore and Hungary, on geometry in France and on estimation in the USA and England. Differences in the use of illustration and colour, decorative in England as opposed to functional in Hungary, have also been identified. The philosophy of textbook construction seems to indicate that, whereas in many countries the teachers' guides focus on developing mathematics, or mathematics education principles, English schemes tend to be management focused, concentrating to a large extent on classroom organisation.

Where textbooks have to be approved, the obvious criterion, coverage of and adherence to the curriculum, is applied. Some systems extend the approval system to include various aspects of clarity and approach and value for money. The power of textbooks is generally acknowledged, but to study this more fully the actual way that textbooks are used in the classroom needs to be better understood.

### **Use of calculators**

Material on actual calculator use in the countries involved in the international review of curriculum and assessment frameworks has been taken from TIMSS (where TIMSS has relevant information. This does not include all 16 countries involved in the current study). Patterns of calculator use for ten- and 14-year-olds are shown in Tables 6 and 7 below.

The systems with a centralised government fall into two distinct types, those where the majority of pupils at age ten use calculators 'never or hardly ever', and those where calculator use is more common, England and New Zealand. These two countries and the federal states show considerable diversity in calculator use.

More data are available on patterns of calculator use at age 14 and, for only two systems, Japan and Korea, do most pupils 'never or hardly ever' use calculators. By this age the most commonly reported pattern for calculator use is 'almost every day' in most of the systems, but many of these still show considerable diversity. Spain and Switzerland represent the greatest diversity with a substantial number of pupils in the extreme categories.

**Table 6: Teachers' reports on frequency of calculator use (Grade 4/Age Ten)**

**TIMSS Data**

(% of pupils)

<b>Country</b>	<b>Never or Hardly Ever</b>	<b>Once or Twice a Month</b>	<b>Once or Twice a Week</b>	<b>Almost Every Day</b>
<b>England</b>	8%	<b>39%</b>	<b>42%</b>	11%
<b>Hungary</b>	<b>78%</b>	9%	2%	12%
<b>Japan</b>	<b>94%</b>	5%	1%	0%
<b>Korea</b>	<b>86%</b>	8%	4%	2%
<b>Netherlands</b>	<b>85%</b>	11%	2%	2%
<b>New Zealand</b>	5%	22%	<b>42%</b>	<b>30%</b>
<b>Singapore</b>	<b>97%</b>	2%	1%	0%
<b>Australia</b>	11%	<b>33%</b>	<b>43%</b>	13%
<b>Canada</b>	<b>37%</b>	<b>35%</b>	<b>25%</b>	4%
<b>USA</b>	<b>29%</b>	<b>32%</b>	<b>28%</b>	11%

**Table 7: Teachers' Reports on frequency of calculator use (Grade 8/Age 14)**

**TIMSS Data**

(% of pupils)

Country	Never or Hardly Ever	Once or Twice a Month	Once or Twice a Week	Almost Every Day
England	0%	2%	15%	<b>83%</b>
France	4%	3%	19%	<b>74%</b>
Hungary	29%	5%	6%	<b>60%</b>
Japan	<b>79%</b>	16%	4%	2%
Korea	<b>76%</b>	16%	8%	1%
Netherlands	0%	2%	17%	<b>81%</b>
New Zealand	7%	5%	21%	<b>66%</b>
Singapore	1%	5%	12%	<b>82%</b>
Spain	<b>40%</b>	4%	11%	<b>45%</b>
Sweden	7%	21%	<b>37%</b>	<b>35%</b>
Australia	6%	1%	10%	<b>83%</b>
Canada	5%	3%	12%	<b>80%</b>
Germany	19%	5%	15%	<b>62%</b>
Switzerland	<b>36%</b>	8%	<b>24%</b>	<b>32%</b>
USA	8%	10%	20%	<b>62%</b>

Taken together, Tables 6 and 7 indicate a lack of consensus on the role of the calculator in most of the 16 countries of the study. Those with the greatest consistency are predominantly Pacific Rim states with centralised governments. The tables also suggest that attributing frequency of calculator use to particular countries, and not merely federal ones, is a hazardous undertaking. The characteristics of the system as a whole may well not be shared by individual schools, and hence classes or pupils.

**Teacher specialisation**

Practice in whether pupils are taught mathematics by generalists or specialists is generally consistent, with the youngest pupils being taught by generalists and 13- and 14-year-olds taught by specialists. The only variations concern the degree to which older primary pupils experience some specialist teaching and whether lower secondary pupils experience any teaching from generalists.

At the lower ages, Germany indicates some specialist teaching from age nine onwards, and Hungary from age 11 onwards. In Italy, there is some specialist teaching in primary schools. In secondary schools in New Zealand, there are some generalists teaching in the first two years of secondary schooling. In Sweden, Singapore, the Netherlands and Victoria (Australia), secondary teachers tend to specialise in two or three subjects. The remaining countries have a straightforward division between generalists in primary schools and specialists in secondary schools.

While the general picture is one of consensus between countries on the use of generalists to teach mathematics in the primary school with increasing teacher specialisation thereafter, the reasons for this are not totally clear. The economics and practicalities of specialist teaching of mathematics in the primary school may account for the dominance of generalist teachers, particularly in states where many primary schools are small. Similarly educational considerations on teacher-child relationships may be an important factor. For older pupils in this age range, the degree of teacher specialisation is the real variable, rather than whether or not teachers specialise. The advantages and disadvantages of specialising in one or two subjects would be interesting to follow up.

## **Conclusions**

A comparative study such as this one, which has looked at mathematics education for the five to 14 age range in the 16 countries of the international review of curriculum and assessment frameworks project, is likely to find both areas where broad consensus exists between countries, and issues on which views are divided. The availability of data on actual practice, for example on time spent teaching mathematics, highlights the dangers of over-generalising the situation presented by official sources.

Any summary of the picture revealed in the study runs the risk of over-simplification, but it is possible, in broad terms, to identify consensus and disagreement, but with a caveat about differences between intended practice and actual practice. The study has shown:

- agreement between states with a centralised government on the usefulness of some form of national curriculum, but disagreement on its form;
- consensus on the broad areas of mathematics which should be included in the curriculum for pupils aged five to 14, but differences in approach, terminology and implementation;
- the tendency for the curriculum to be spiral in nature, although the frequency of teaching of particular topics in the classroom is difficult to establish. However, textbook comparisons reveal interesting differences here;
- broad agreement on the proportion of the available teaching time to be devoted to mathematics, but differences in the actual time spent, and considerable diversity between schools in many countries where data on time spent teaching mathematics are available for scrutiny;
- diversity in assessment arrangements arising from differences in purpose, and the degree to which countries find it possible, or desirable, to make assessment systems compulsory;

- disagreement as to the degree of differentiation of the curriculum, resulting from differences in view on the balance between entitlement and matching the mathematics to the attainment level of the pupil;
- lack of consensus on the role of the calculator in mathematics education allied to considerable diversity in practice in the classroom in many countries;
- consistent practice in mathematics being taught by generalist teachers to younger pupils in the five to 14 age range but by specialists, although varying in their degree of specialisation, to older pupils.

In addition, it is apparent that the format of a curriculum, the absence or presence of overt performance standards, the frequency of teaching particular mathematical topics, a country's approach to differentiation and its assessment arrangements are inter-related in a complex way which derives both from a series of value judgements on purpose and features of the structure of individual systems.

## Appendix 1

### Sources

The main sources for this report were:

- Material from the *Archive* of the international review of curriculum and assessment frameworks on the 16 countries studied in the project;
- Responses to questionnaires sent to the 16 countries of the study (see Appendix 2), asking them to confirm information on mathematics taken from the *Archive* and requesting additional information on mathematics in response to the questionnaire;
- Information given, and views expressed, at the International Seminar on Mathematics in the School Curriculum, ages five to 14 hosted by QCA in February 1998 (see Appendices 3 and 4);
- Reports of study visits to Japan and Singapore by QCA officers;
- Material from the Third International Mathematics and Science Study (TIMSS):
  - BEATON, A. et al. (1996). *Mathematics Achievement in the Middle School Years, IEA's Third International Mathematics and Science Study*. Boston: Center for the Study of Testing, Evaluation and Educational Policy, Boston College.
  - MULLIS, I. et al. (1997). *Mathematics Achievement in the Primary School Years, IEA's Third International Mathematics and Science Study*. Boston: Center for the Study of Testing, Evaluation and Educational Policy, Boston College.

**Appendix 2 - Table 1: Outline of the Education Systems in the 16 Countries of the International Review of Curriculum and Assessment Frameworks Project**

	Eng	<i>Aust</i>	<i>Can</i>	Fran	<i>Germ</i>	Hung	Italy	Japan	Korea	Neth	N
<i>Compulsory education</i>											
Starting age	5	<i>5/6</i>	<i>6</i>	6	6	6 <sup>1</sup>	6	6	6	5	6
Minimum school leaving age	16	<i>15/16</i>	<i>15/16</i>	16 <sup>3</sup>	<i>18<sup>4</sup></i>	16 <sup>5</sup>	14+	15	15	16 <sup>6</sup>	1
Duration in years	11	<i>10/11</i>	<i>9-10</i>	10	<i>10 FT +2 PT</i>	11	8	9	9	10 FT +2 PT	1
<i>Educational phases (not necessarily involving transfer from one school to another)</i>											
Pre-school	2-5	<i>3-5</i>	<i>5</i>	3-6	<i>3-6</i>	3-6 5 comp	3-6	3-6	3-6	4-5	3
Primary/basic	5-11	<i>5/6 to 12/13</i>	<i>5 to 12/13</i>	6-11	<i>6 to 10/12</i>	6 to 10/14	6-11	6-12	6-12	5-12	5
Lower secondary	11-16	<i>12/13-15/16</i>	<i>12/13 - 17/18</i>	11-15	<i>10/12 to 16</i>	10/14- to 16	11-14	12-15	12-15 <sup>8</sup>	12-15	1
Upper secondary	16-18	<i>15/16 to 18</i>	<i>17-18+</i>	15-18	<i>16 to 18/19</i>	16-18	14-18	15-18	15-18	15 to 16/18	1
	Eng	<i>Aust</i>	<i>Can</i>	Fran	<i>Germ</i>	Hung	Italy	Japan	Korea	Neth	N

Ages overlap as categories show the age of most pupils at start and end of each school phase.

The countries in italics are those with devolved responsibility for education. The information provided, particularly for these countries, may not apply to all regions.

<sup>1</sup> **Hungary:** But attendance in pre-compulsory education at age five is also compulsory.

<sup>2</sup> **Sweden:** There are proposals to extend the starting age to six for all students from September 1998.

<sup>3</sup> **France:** Most students complete lower secondary education at 15 and are consequently required to continue into upper secondary education until they are aged 16.

<sup>4</sup> **Germany:** Students must complete at least nine to ten years of full-time education followed by two to three years of part-time education.

<sup>5</sup> **Hungary:** There is a proposal to extend this to age 18 from 1998.

<sup>6</sup> **Netherlands:** Full-time from age five to 17 OR full-time from five to 16 plus two years' part-time education. After age 16, school fees become payable on a means-tested basis.

<sup>7</sup> **Singapore:** The average of ten years' formal general education is *universal but not compulsory*

<sup>8</sup> **Korea:** The first cycle of secondary education (age 12-15) is gradually becoming compulsory throughout the country. Post-compulsory schools (15-18) charge fees.